$$Find: \left[\begin{array}{cc} a & b \\ c & d \end{array}\right]^{-1}$$

$$\left[\begin{array}{cc|cc} a & b & 1 & 0 \\ c & d & 0 & 1 \end{array}\right] \ R_1 \Leftarrow \left(\frac{1}{a}\right) R_1$$

$$\left[\begin{array}{cc|cc} 1 & \frac{b}{a} & \frac{1}{a} & 0 \\ c & d & 0 & 1 \end{array}\right] \ R_2 \Leftarrow R_2 + (-c) R_1$$

$$\left[\begin{array}{cc|cc} 1 & \frac{b}{a} & \frac{1}{a} & 0 \\ 0 & d + \frac{-bc}{a} & -\frac{c}{a} & 1 \end{array}\right] \ aside: d + \frac{-bc}{a} = \frac{ad}{a} - \frac{bc}{a} = \frac{ad-bc}{a}$$

$$\left[\begin{array}{cc|cc} 1 & \frac{b}{a} & \frac{1}{a} & 0 \\ 0 & \frac{ad-bc}{a} & -\frac{c}{a} & 1 \end{array}\right] \ R_2 \Leftarrow \left(\frac{a}{ad-bc}\right) R_2$$

$$\left[\begin{array}{cc|cc} 1 & \frac{b}{a} & \frac{1}{a} & 0 \\ 0 & 1 & \frac{-c}{ad-bc} & \frac{a}{ad-bc} \end{array}\right] \ R_1 \Leftarrow R_1 + \left(\frac{-b}{a}\right) R_2$$

$$\left[\begin{array}{cc|cc} 1 & 0 & \left(\frac{1}{a} + \frac{-b}{a} \cdot \frac{-c}{ad-bc}\right) & \left(\frac{-b}{a} \cdot \frac{a}{ad-bc}\right) \\ 0 & 1 & \frac{-c}{ad-bc} & \frac{a}{ad-bc} \end{array}\right] aside: \begin{array}{c} \left(\frac{ad-bc}{ad-bc}\right) \cdot \left(\frac{1}{a} + \frac{-b}{a} \cdot \frac{-c}{ad-bc}\right) \\ = \frac{(ad-bc)+(bc)}{(ad-bc)\cdot(a)} \\ = \frac{ad}{(a)\cdot(ad-bc)} \\ = \frac{d}{ad-bc} \end{array}$$

$$\left[\begin{array}{cc|cc} 1 & 0 & \frac{d}{ad-bc} & \frac{-b}{ad-bc} \\ 0 & 1 & \frac{-c}{ad-bc} & \frac{a}{ad-bc} \end{array}\right]$$

$$\left[\begin{array}{cc} a & b \\ c & d \end{array}\right]^{-1} == \frac{1}{ad-bc} \left[\begin{array}{cc} d & -b \\ -c & a \end{array}\right]$$